# Multiplicity Uncertainty Analysis for the Hage-Cifarelli Formulism

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# Overview

- Simple overview of the Hage formulism
  - Equations
  - Solutions
- Uncertainty in the equations
  - Calculate and display the uncertainty

# Calculational Side

$$R_1\left(\varepsilon, M, F_S, F_S'\right) = \varepsilon M\left[\overline{\upsilon_{S1}} F_S + \overline{\upsilon_{S1}'} F_S'\right]$$

$$R_2\left(\varepsilon, M, F_S, F_S'\right) = \left(\varepsilon M\right)^2\left[b_{21} F_S + b_{21}' F_S'\right]$$

$$b_{21} = \overline{\upsilon_{S2}} + \frac{M-1}{\overline{\upsilon_{I1}}-1}\overline{\upsilon_{S1}}\,\overline{\upsilon_{I2}} \qquad b_{21}' = \overline{\upsilon_{S2}'} + \frac{M-1}{\overline{\upsilon_{I1}}-1}\overline{\upsilon_{S1}'}\,\overline{\upsilon_{I2}}$$

$$R_3\left(\varepsilon, M, F_S, F_S'\right) = \left(\varepsilon M\right)^3\left[b_{31} F_S + b_{31}' F_S'\right]$$

$$b_{31} = \overline{\upsilon_{S3}} + \frac{M-1}{\overline{\upsilon_{I1}}-1}\left[\overline{\upsilon_{S1}}\,\overline{\upsilon_{I3}} + 2\overline{\upsilon_{S2}}\,\overline{\upsilon_{I2}}\right] + 2\left(\frac{M-1}{\overline{\upsilon_{I1}}-1}\right)^2 \overline{\upsilon_{S1}}\left(\overline{\upsilon_{I2}}\right)^2$$

$$b_{31}' = \overline{\upsilon_{S3}'} + \frac{M-1}{\overline{\upsilon_{I1}}-1}\left[\overline{\upsilon_{S1}'}\,\overline{\upsilon_{I3}} + 2\overline{\upsilon_{S2}'}\,\overline{\upsilon_{I2}}\right] + 2\left(\frac{M-1}{\overline{\upsilon_{I1}}-1}\right)^2 \overline{\upsilon_{S1}'}\left(\overline{\upsilon_{I2}}\right)^2$$

# Measured Side

$$R_1\left(m_1,\tau\right)=\frac{m_1}{\tau}$$

$$R_2\left(m_1,m_2,\tau\right)=\frac{m_2-\frac{1}{2}\left(m_1^2\right)}{\tau\omega_2}$$

$$\omega_2=1-\frac{1-e^{-\lambda\tau}}{\lambda\tau}$$

$$R_3\left(m_1,m_2,m_3,\tau\right)=\frac{m_3-m_2m_1+\frac{1}{3}\left(m_1\right)^3}{\tau\omega_3}$$

$$\omega_3=1-\frac{3-4e^{-\lambda\tau}+e^{-2\lambda\tau}}{2\lambda\tau}$$

# Solution is pretty easy

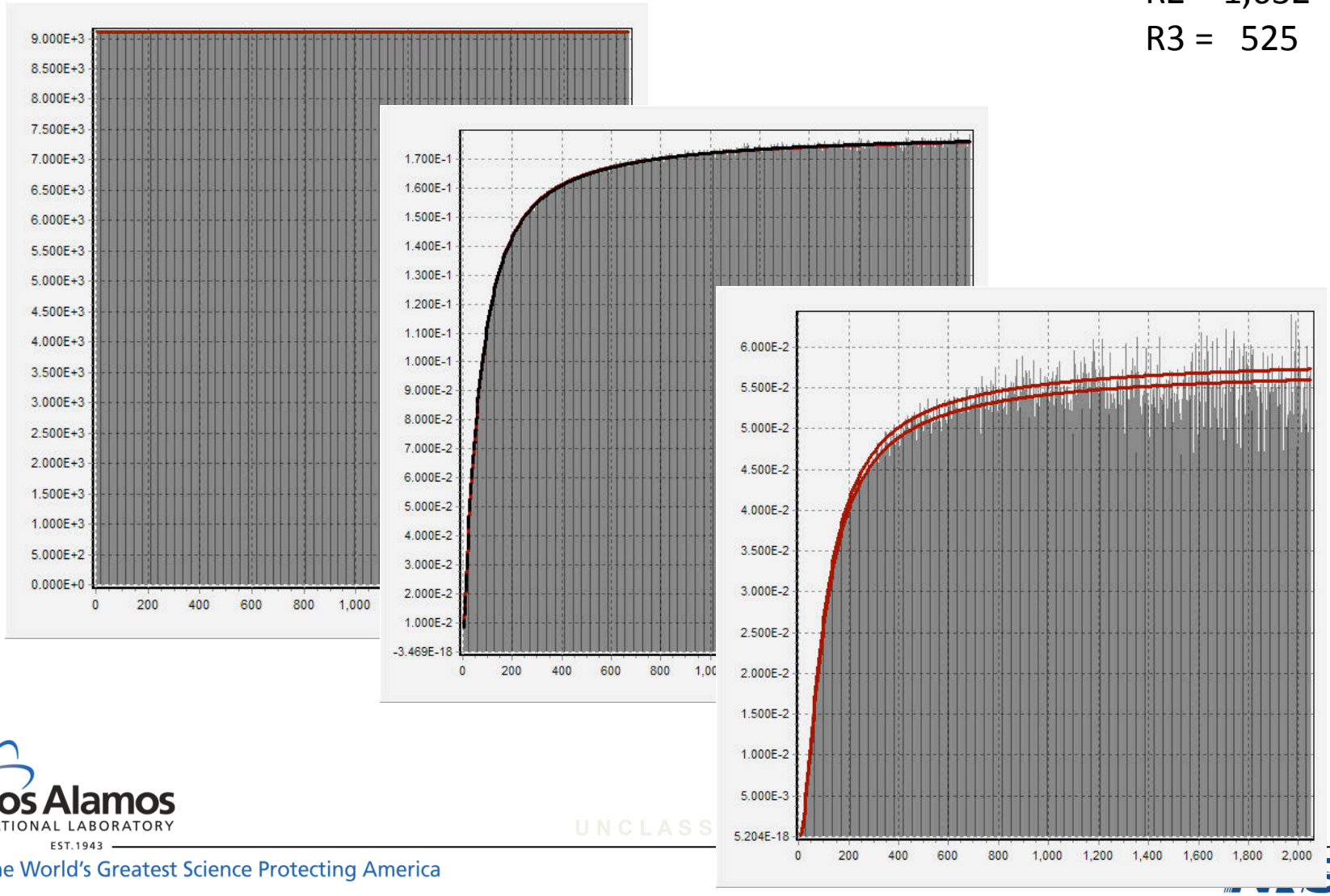
$$R_1\left(\varepsilon, M, F_S, F_S'\right) = R_1\left(m_1, \tau\right) = R_1\left(\tau \to \infty\right)$$

$$R_2\left(\varepsilon, M, F_S, F_S'\right) = R_2\left(m_1, m_2, \tau\right) = R_2\left(\tau \to \infty\right)$$

$$R_3\left(\varepsilon, M, F_S, F_S'\right) = R_3\left(m_1, m_2, m_3, \tau\right) = R_3\left(\tau \to \infty\right)$$

# Bare BeRP Ball

R1 = 9,104
R2 = 1,632
R3 = 525



# Bare BeRP Ball

```
restart :
R1 := (eff, M, Fs, Fsp) → eff·M·(vs1·Fs + vs1p·Fsp);
R2 := (eff, M, Fs, Fsp) → (eff·M)^2·(b21(M)·Fs + b21p(M)·Fsp);
R3 := (eff, M, Fs, Fsp) → (eff·M)^3·(b31(M)·Fs + b31p(M)·Fsp);
```

$$(eff, M, Fs, Fsp) \rightarrow eff\, M\, (vs1\, Fs + vs1p\, Fsp)$$

$$(eff, M, Fs, Fsp) \rightarrow eff^2\, M^2\, (b21(M)\, Fs + b21p(M)\, Fsp)$$

$$(eff, M, Fs, Fsp) \rightarrow eff^3\, M^3\, (b31(M)\, Fs + b31p(M)\, Fsp) \quad (1)$$

```
b21 := (M) → vs2 + (M − 1)/(vi1 − 1)·vs1·vi2;
b21p := (M) → vs2p + (M − 1)/(vi1 − 1)·vs1p·vi2;
b31 := (M) → vs3 + (M − 1)/(vi1 − 1)·(vs1·vi3 + 2·vs2·vi2) + 2·((M − 1)/(vi1 − 1))^2·vs1·vi2^2;
b31p := (M) → vs3p + (M − 1)/(vi1 − 1)·(vs1p·vi3 + 2·vs2p·vi2) + 2·((M − 1)/(vi1 − 1))^2·vs1p·vi2^2;
```

$$M \rightarrow vs2 + \frac{(M-1)\, vs1\, vi2}{vi1 - 1}$$

$$M \rightarrow vs2p + \frac{(M-1)\, vs1p\, vi2}{vi1 - 1}$$

$$M \rightarrow vs3 + \frac{(M-1)\, (vs1\, vi3 + 2\, vs2\, vi2)}{vi1 - 1} + \frac{2\, (M-1)^2\, vs1\, vi2^2}{(vi1 - 1)^2}$$

$$M \rightarrow vs3p + \frac{(M-1)\, (vs1p\, vi3 + 2\, vs2p\, vi2)}{vi1 - 1} + \frac{2\, (M-1)^2\, vs1p\, vi2^2}{(vi1 - 1)^2} \quad (2)$$

```
vs1 := 2.153;
vs2 := 1.904;
vs3 := 0.879;
vs1p := 1.0;
vs2p := 0.0;
vs3p := 0.0;
vi1 := 2.879;
vi2 := 3.388;
vi3 := 2.109;
R1t := 9103.691;
R2t := 0.179254·R1t;
R3t := 0.057689·R1t;
Fspt := 194000;
efft := 0.01;
Mt := 3.44;
0.75·vs1/vs1p
```

2.153
1.904
0.879
1.0
0.
0.
2.879
3.388
2.109
9103.691
1631.873027
525.1828301
194000
0.01
3.44
1.614750000 (3)

```
solve({R1(eff, M, Fs, 0.75·Fs·vs1/vs1p) = R1t, R2(eff, M, Fs, 0.75·Fs·vs1/vs1p) = R2t, R3(eff, M, Fs, 0.75·Fs·vs1/vs1p) = R3t}, {eff, M, Fs});
```

{Fs = −2564.864728, M = 0.6142041057, eff = −1.533762974}, {Fs = 28136.30240, M = 1.877402864, eff = 0.04574155567} (4)

```
solve({R1(efft, M, Fs, Fsp) = R1t, R2(efft, M, Fs, Fsp) = R2t, R3(efft, M, Fs, Fsp) = R3t}, {M, Fs, Fsp});
```

{Fs = 1.159335055 10^5, Fsp = 15529.59693, M = 3.433613224}, {Fs = −8.621167459 10^5 − 133.5515843 I, Fsp = 1.856139863 10^6 + 144.8230350 I, M = 112.1128829 + 6377.025590 I}, {Fs = 73251.92204, Fsp = −5.077089012 10^5, M = −2.601073054}, {Fs = −8.658210625 10^5, Fsp = 1.860020621 10^6, M = −222.4684827}, {Fs = −8.621167459 10^5 + 133.5515843 I, Fsp = 1.856139863 10^6 − 144.8230350 I, M = 112.1128829 − 6377.025590 I} (5)

```
solve({R1(eff, Mt, Fs, Fsp) = R1t, R2(eff, Mt, Fs, Fsp) = R2t, R3(eff, Mt, Fs, Fsp) = R3t}, {eff, Fs, Fsp});
```

{Fs = 1.164036079 10^5, Fsp = 15169.60601, eff = 0.009956943136}, {Fs = 8988.700175, Fsp = −23177.27495, eff = −0.6919467127} (6)

```
solve({R1(eff, M, Fs, Fspt) = R1t, R2(eff, M, Fs, Fspt) = R2t, R3(eff, M, Fs, Fspt) = R3t}, {eff, M, Fs});
```

{Fs = −23085.45410 + 1.172583085 10^5 I, M = 2.247679452 + 2.472822545 I, eff = −0.002891911747 − 0.008911166410 I}, {Fs = −90106.81266 − 7.460826558 I, M = 12.42713705 + 5923.457514 I, eff = 0.09567741179 + 0.000008025027371 I}, {Fs = −92017.35217, M = −22.66728619, eff = 0.09763855490}, {Fs = −90106.81266 + 7.460826558 I, M = 12.42713705 − 5923.457514 I, eff = 0.09567741179 − 0.000008025027371 I}, {Fs = −23085.45410 − 1.172583085 10^5 I, M = 2.247679452 (7)

# Bare BeRP Ball

| Fixed | Eff | MT | ML | Fs | Fs’ | Alpha ratio |
|---|---|---|---|---|---|---|
| Fs’ | 0.0085 | 5.1 | 3.7 | 135,000 | 0 | 0 |
| Eff | 0.0095 | 4.8 | 3.5 | 121,000 | 12,000 | 0.05 |
| ML | 0.011 | 4.4 | 3.2 | 99,000 | 27,000 | 0.13 |
| Fs | 0.0090 | 5.0 | 3.6 | 128,000 | 6,000 | 0.02 |
| | | | | | | |
| “Truth” | ~0.0095 | ~4.4 | ~3.2 | ~128,000 | ~0 | ~0 |

Red font indicates fixed value

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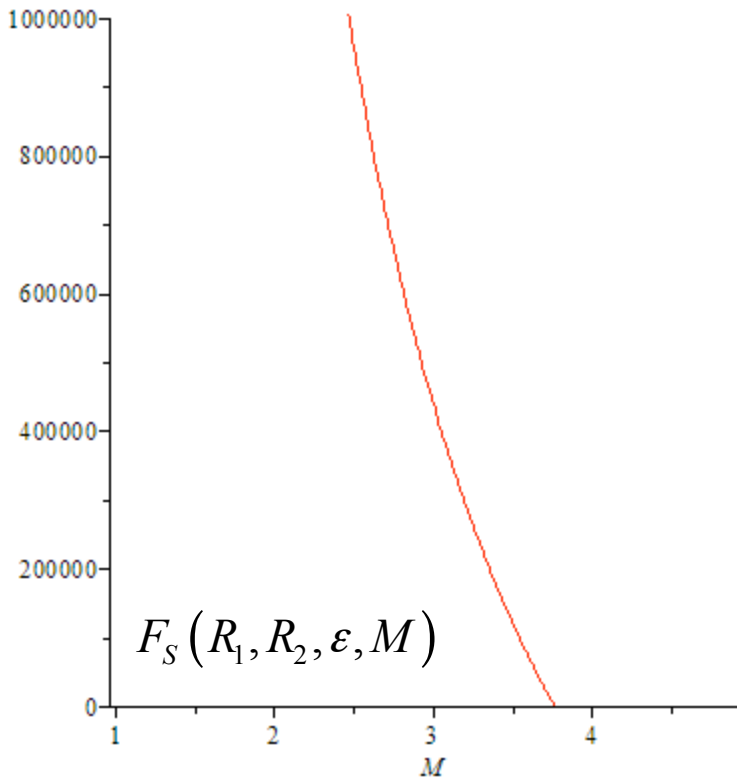
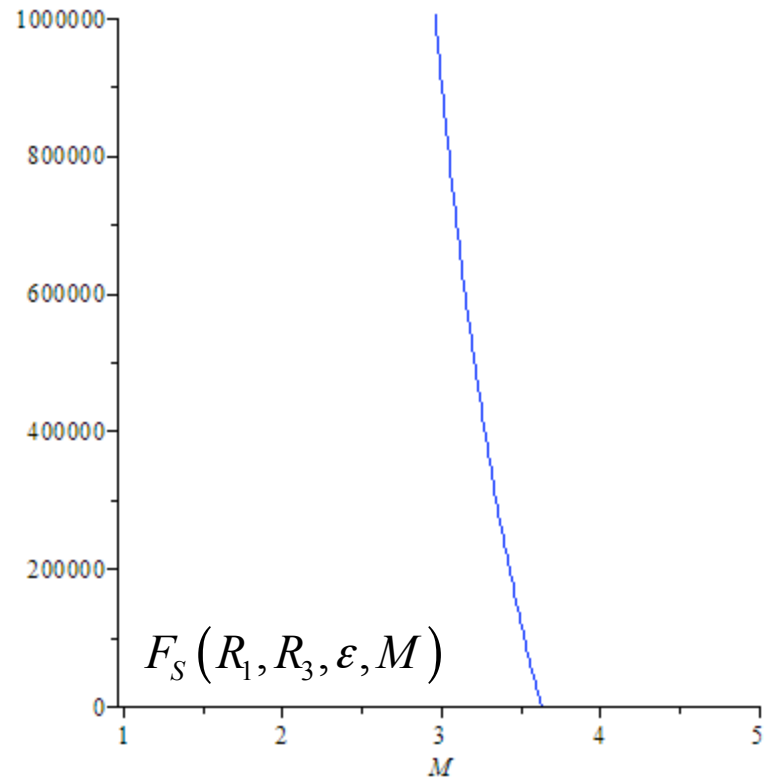
# Bare BeRP Ball




$\varepsilon = 0.0095$

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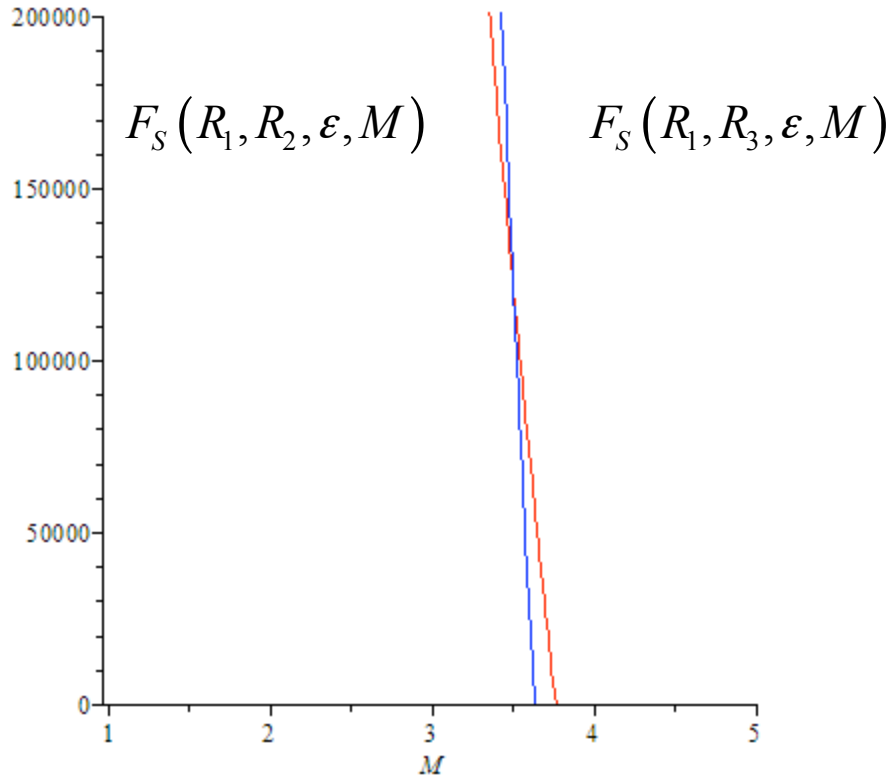
# What is the solution




$\varepsilon = 0.0095$

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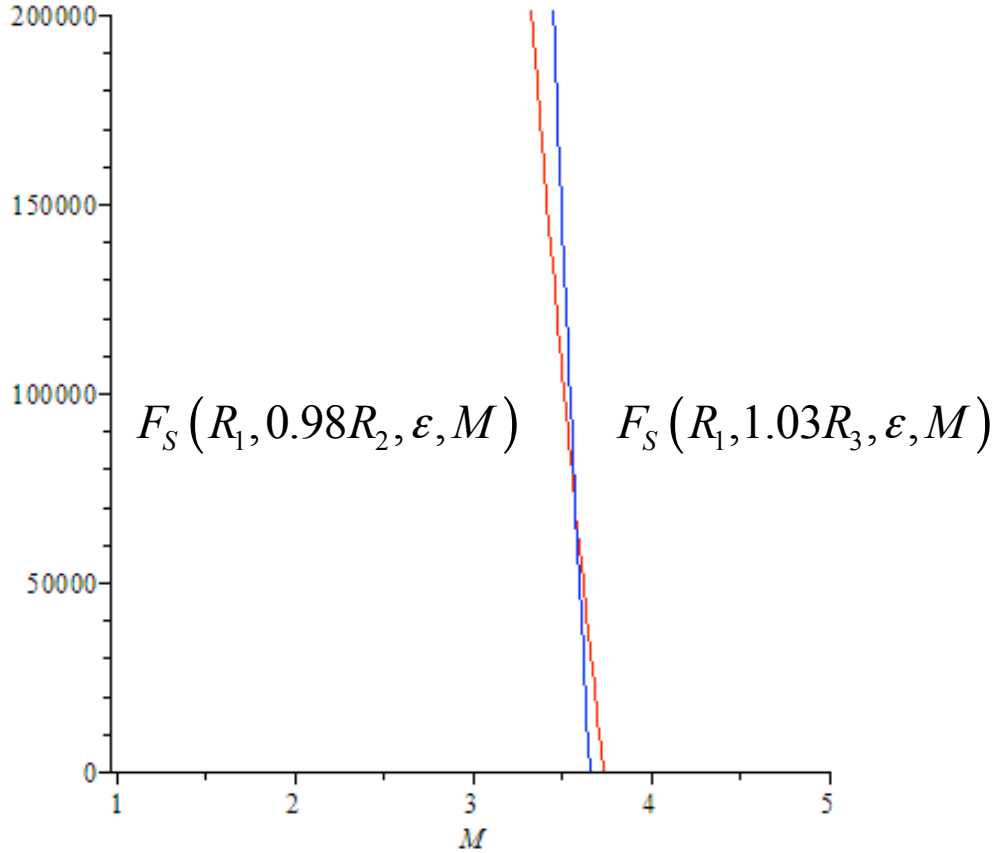
# How does the solution change




$\varepsilon = 0.0095$

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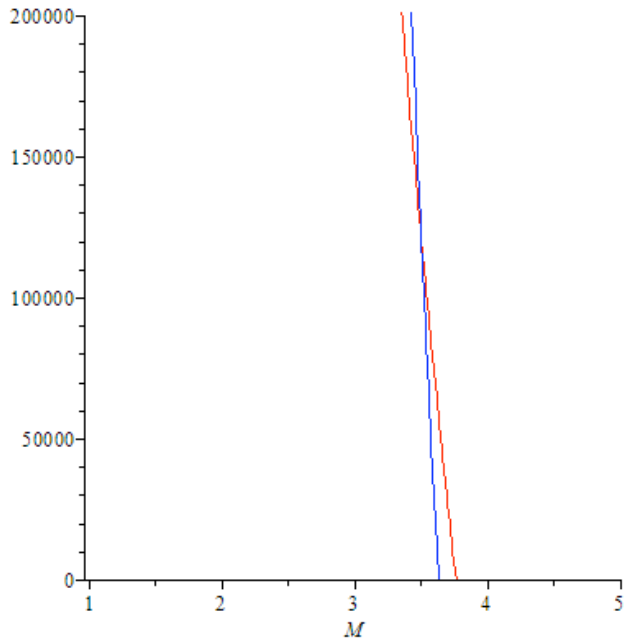


$$\chi^2 = \frac{\left(R_1\left(\varepsilon, M, F_S, F_S'\right) - R_1\left(\tau \to \infty\right)\right)^2}{\Delta R_1\left(\tau \to \infty\right)} + \frac{\left(R_2\left(\varepsilon, M, F_S, F_S'\right) - R_2\left(\tau \to \infty\right)\right)^2}{\Delta R_2\left(\tau \to \infty\right)} + \frac{\left(R_3\left(\varepsilon, M, F_S, F_S'\right) - R_3\left(\tau \to \infty\right)\right)^2}{\Delta R_3\left(\tau \to \infty\right)}$$

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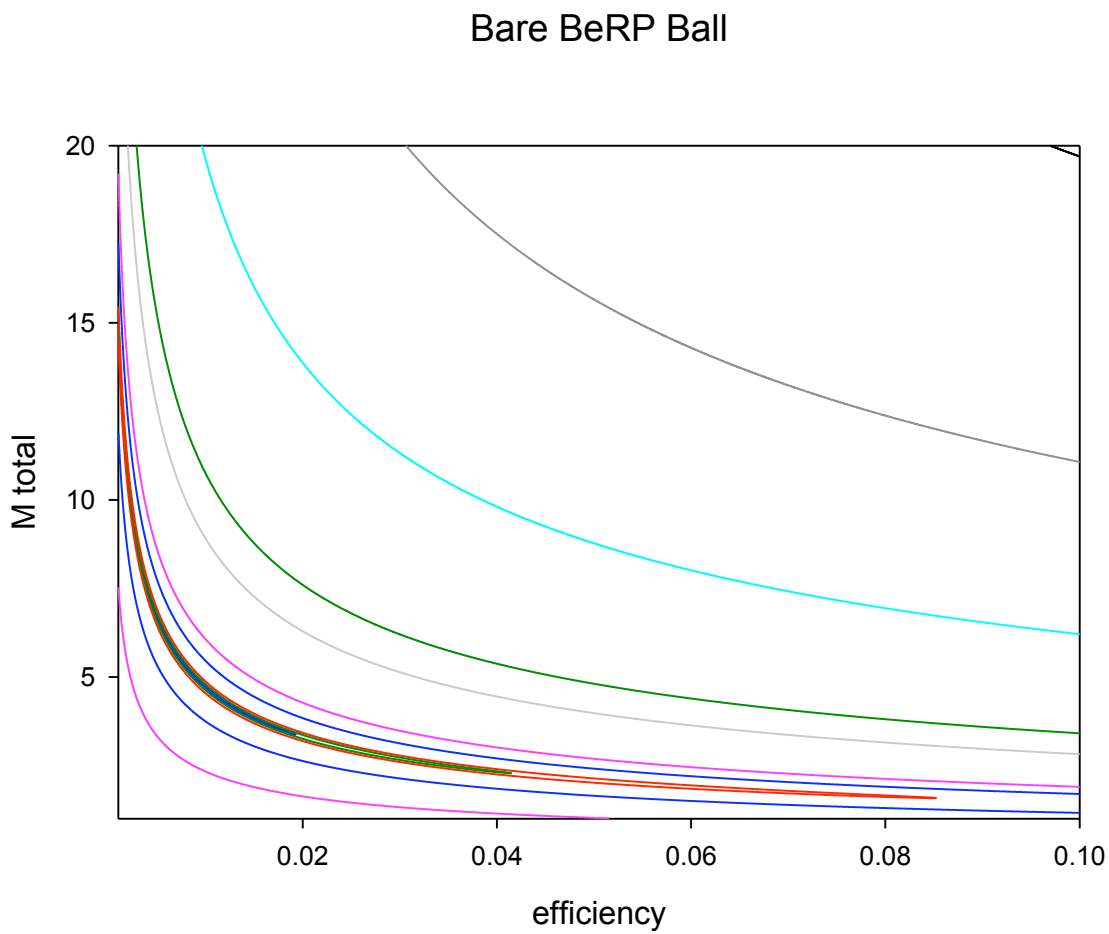
# What is the uncertainty?




Assumed alpha = 0

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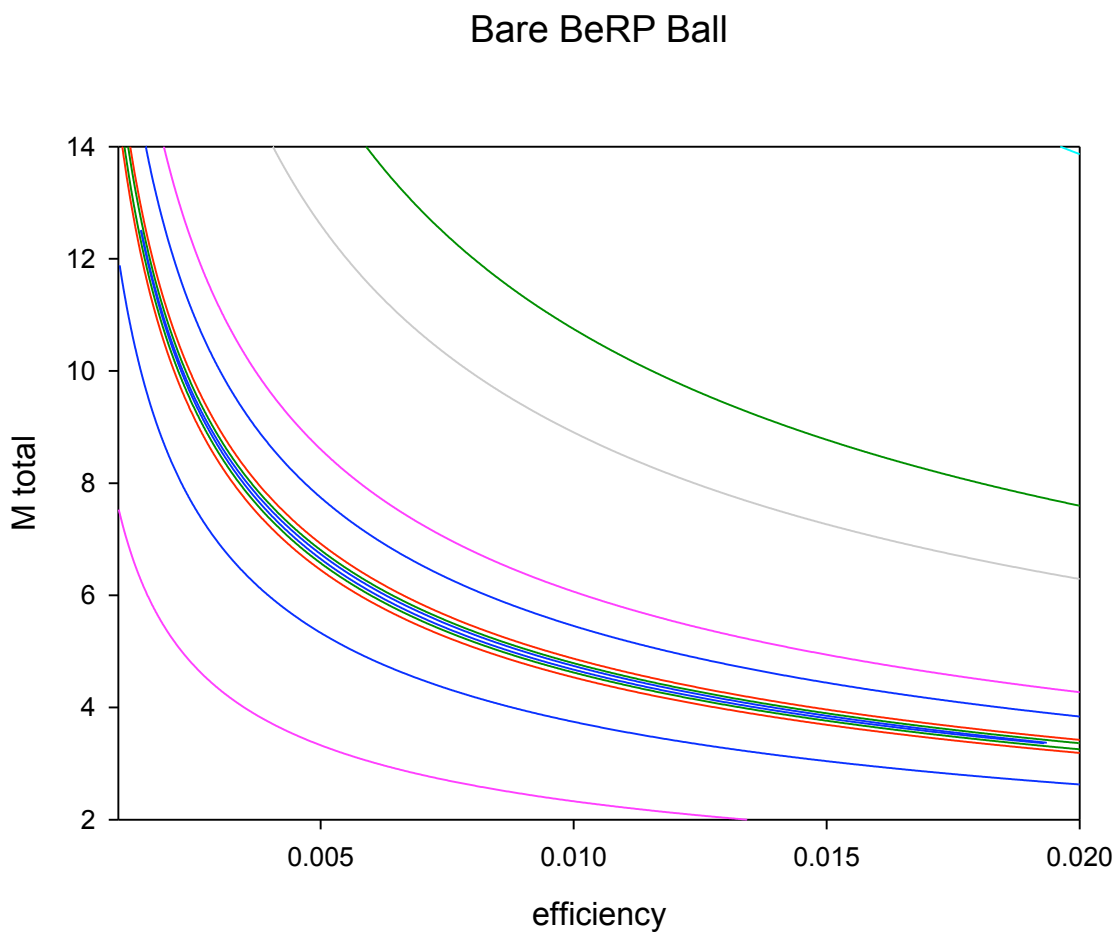


Assumed alpha = 0

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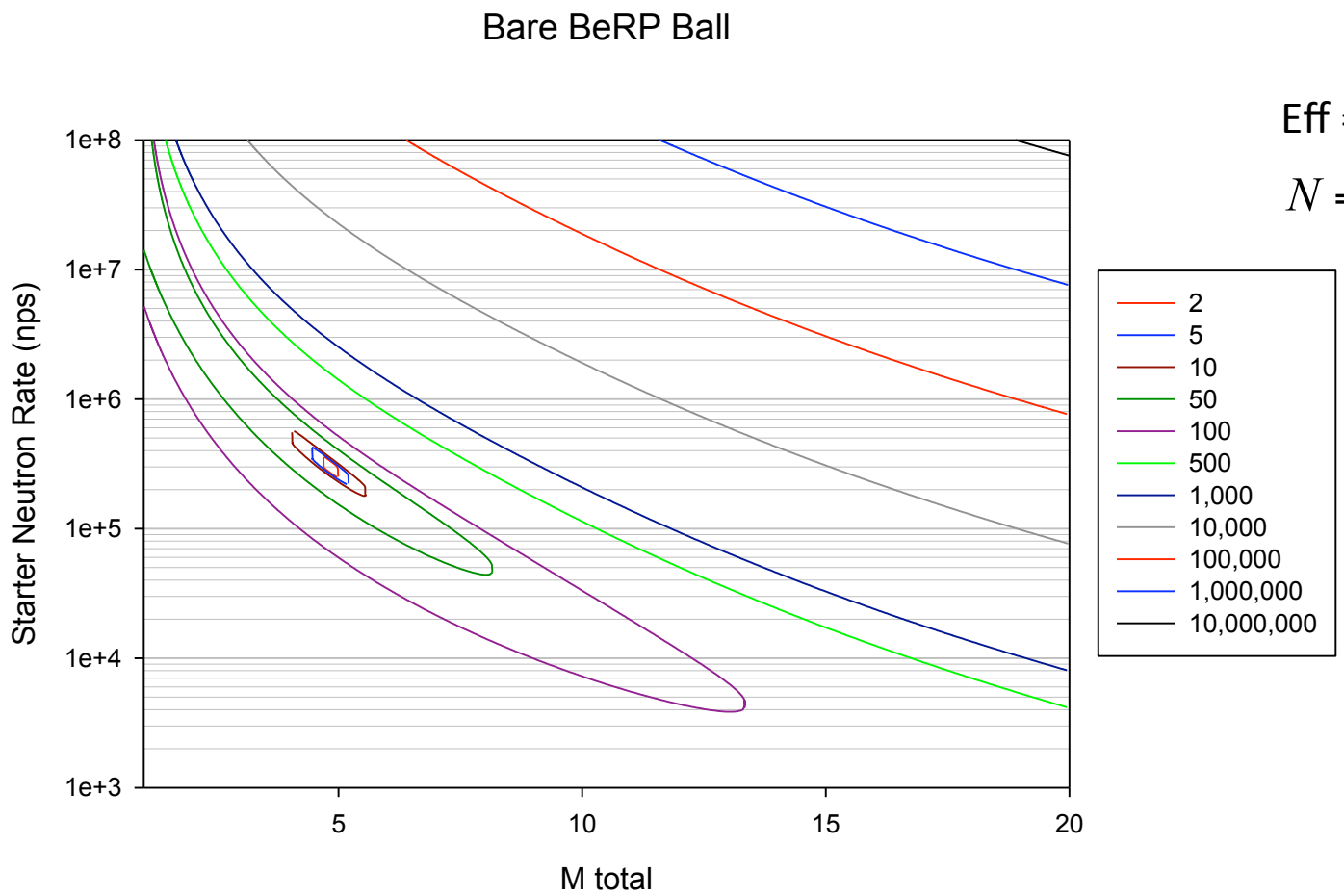
# M vs N




Eff = 0.0095

$$N = \overline{\upsilon_{S1}} F_S + F_S'$$

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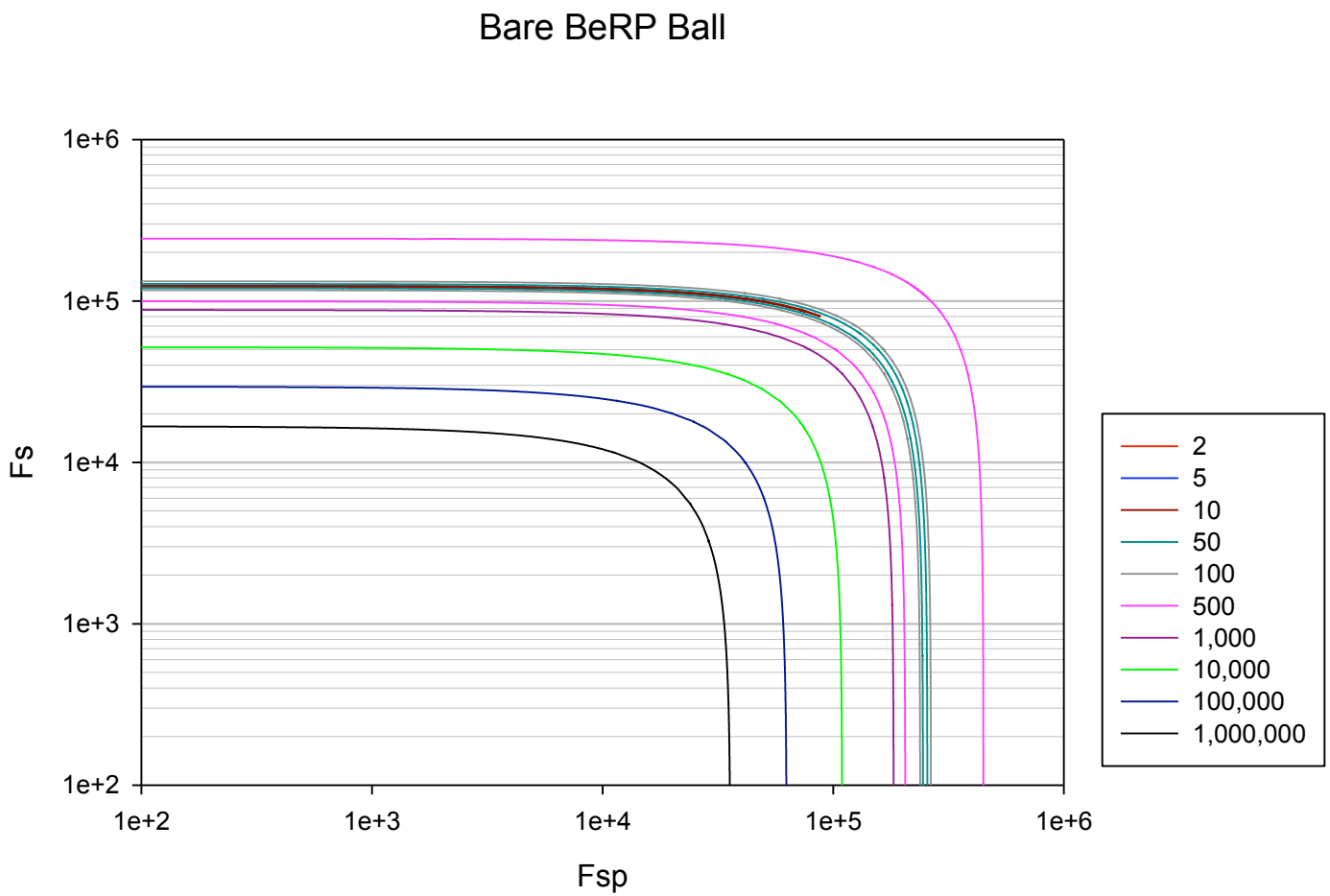
# Fs vs F's

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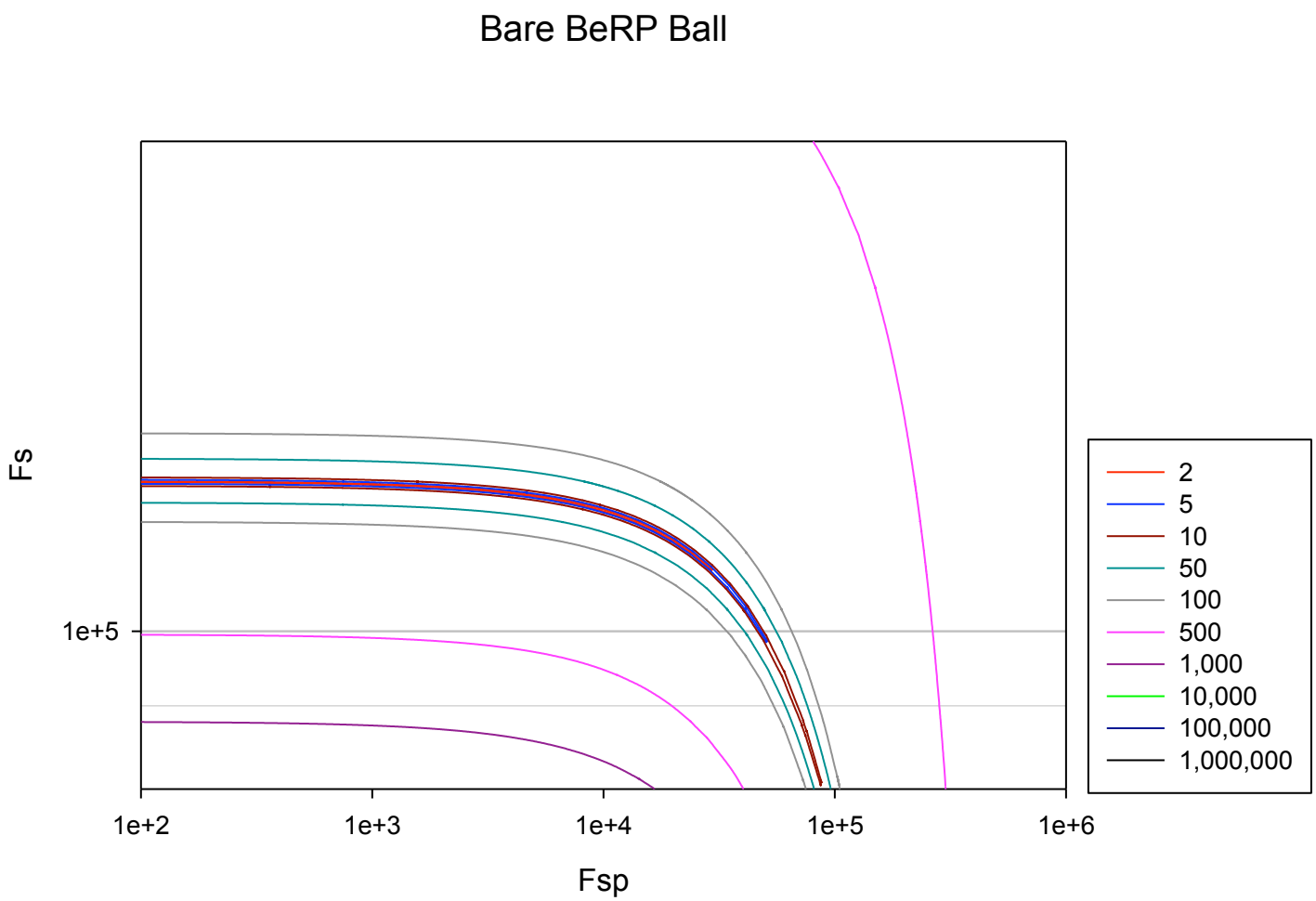
# Fs vs F's

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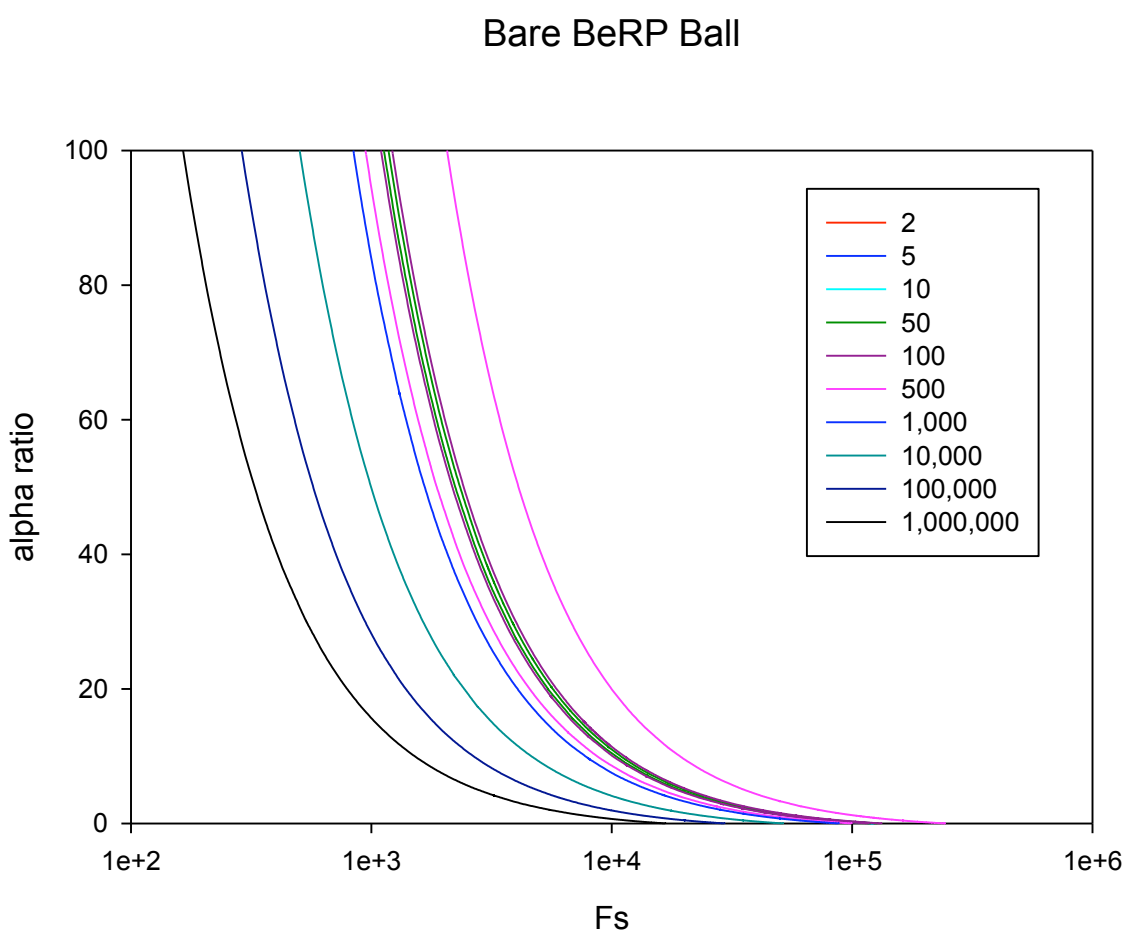
# Fs vs alpha ratio



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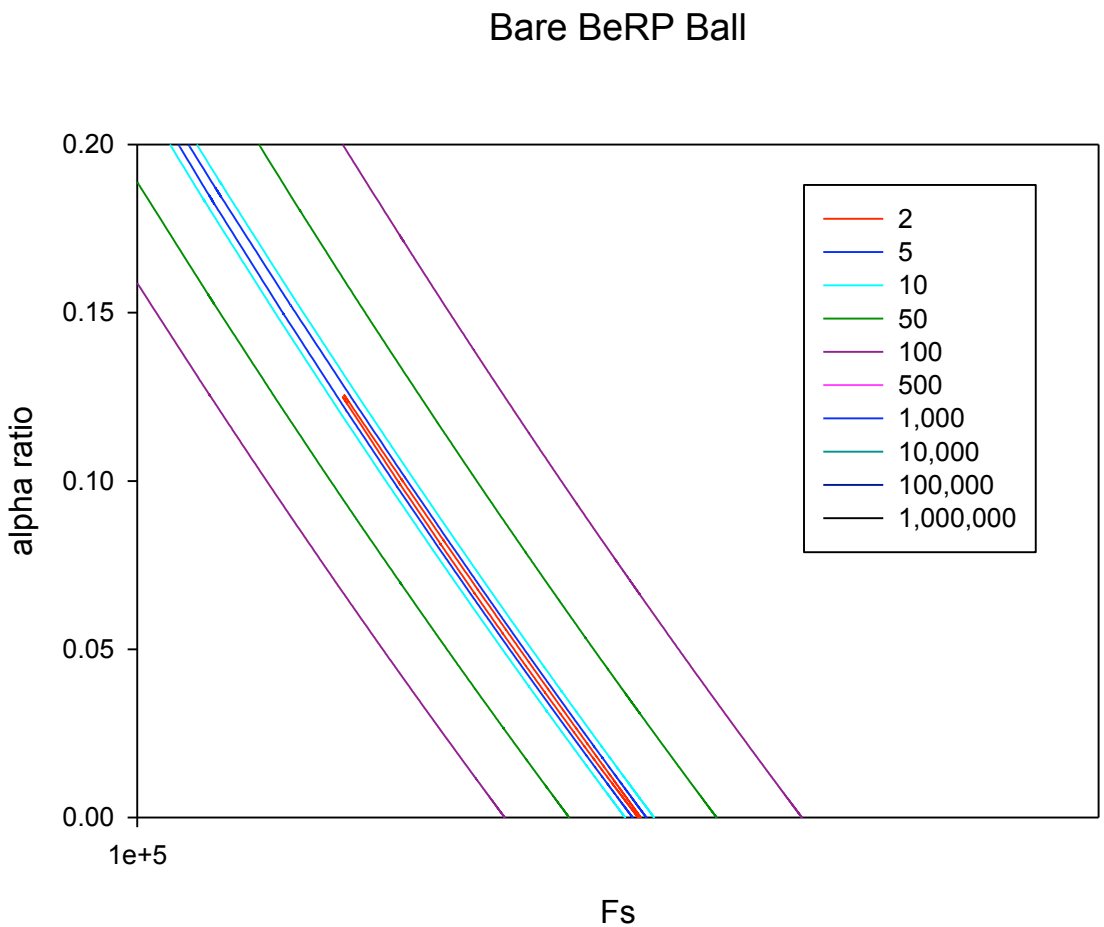
# Fs vs alpha ratio

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# Solutions for the bare BeRP ball

| | $\chi^2=2$ | $\chi^2=5$ | $\chi^2=10$ |
|---|---|---|---|
| $M_{Total}$ | 4.7 → 5.0 | 4.5→5.2 | 4.0→5.5 |
| $N = \overline{\upsilon_{S1}} F_S + F_S'$ | 250,000→360,000 | 220,000→423,000 | 179,000→567,000 |
| $\alpha = \frac{F_S'}{\overline{\upsilon_{S1}} F_S}$ | 0→0.12 | 0→0.24 | 0→0.50 |
| Assuming efficiency = 0.0095 | | | |

# Degenerate Example

$$y = \frac{1}{km} \qquad x = \frac{1}{(km)^2}$$

$$m = \frac{1}{yk}$$

$$x = \frac{1}{\left(k\frac{1}{yk}\right)^2} = y^2$$

# Is the 2nd moment degenerate

$$R_1\left(\varepsilon, M, F_S, F_S'\right) = \varepsilon M\left[\overline{\upsilon_{S1}}F_S + \overline{\upsilon_{S1}'}F_S'\right]$$

$$R_2\left(\varepsilon, M, F_S, F_S'\right) = \left(\varepsilon M\right)^2\left[b_{21}F_S + b_{21}'F_S'\right] \qquad b_{21} = \overline{\upsilon_{S2}} + \frac{M-1}{\overline{\upsilon_{I1}}-1}\overline{\upsilon_{S1}}\,\overline{\upsilon_{I2}}$$

$$R_3\left(\varepsilon, M, F_S, F_S'\right) = \left(\varepsilon M\right)^3\left[b_{31}F_S + b_{31}'F_S'\right] \qquad b_{31} = \overline{\upsilon_{S3}} + \frac{M-1}{\overline{\upsilon_{I1}}-1}\left[\overline{\upsilon_{S1}}\,\overline{\upsilon_{I3}} + 2\overline{\upsilon_{S2}}\,\overline{\upsilon_{I2}}\right] + 2\left(\frac{M-1}{\overline{\upsilon_{I1}}-1}\right)^2\overline{\upsilon_{S1}}\left(\overline{\upsilon_{I2}}\right)^2$$

Let's assume F's = 0

$$M = \frac{R_1}{\varepsilon\overline{\upsilon_{S1}}F_S} \qquad R_2 = \left(\varepsilon M\right)^2\left[\overline{\upsilon_{S2}} + \frac{M-1}{\overline{\upsilon_{I1}}-1}\overline{\upsilon_{S1}}\,\overline{\upsilon_{I2}}F_S\right]$$

$$R_2 = \left(\frac{R_1}{\overline{\upsilon_{S1}}F_S}\right)^2\left[\overline{\upsilon_{S2}} + \frac{R_1\overline{\upsilon_{I2}}}{\left(\overline{\upsilon_{I1}}-1\right)\varepsilon} - \frac{\overline{\upsilon_{S1}}\,\overline{\upsilon_{I2}}F_S}{\overline{\upsilon_{I1}}-1}\right]$$